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Composite relation for laminar free convection in inclined channels with uniform heat flux boundaries

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ABSTRACT

A composite correlation of the average Nusselt number and the channel Rayleigh number for buoyant air flow through inclined channels with uniform heat flux boundaries is presented. The form of the correlation is based on dimensional analysis and is a superposition of the developing and fully developed flow limits. In the limit of fully developed flow, an analytical solution for the Nusselt number is derived. The developing flow limit follows the format of the correlation for a single plate. The composite relationship based on the top wall temperature is $\overline{\mathrm{Nu}} = \left[\frac{6.25(1+r)}{\mathrm{Ra}''\sin\phi} + \frac{1.64}{(\mathrm{Ra}''\sin\phi)^{2/5}}\right]^{-1/2}$, where r is ratio of the heat flux at the top and bottom wall. At inclination angles of $30^\circ \le \phi \le 90^\circ$, this correlation predicts the available data base for $10 \le \mathrm{Ra}'' \le 10^5$ and agrees with the analytical solution for $1 \le \mathrm{Ra}'' \le 10^2$.

1. Introduction

Prior studies of laminar free convection of air in open-ended vertical channels with constant heat flux boundaries [1–10] show the Nusselt number lies between two limits associated with the channel length-to-spacing aspect ratio. At large aspect ratio, the flow becomes fully developed. At small aspect ratio, the flow is developing. At intermediate aspect ratios, Bar-Cohen and Rosenhow [5] show the Nusselt number, based on the temperature at the mid-point along the length of the channel, can be approximated by the composite relationship:

$$\mathrm{Nu}_{1/2} = \Bigg[\overbrace{(c_1\mathrm{Ra}''^{1/2})^{-2}}^{\text{fully developed flow}} + \overbrace{(0.73\mathrm{Ra}''^{1/5})^{-2}}^{\text{developing flow}}\Bigg]^{-1/2}. \quad (1)$$

For fully developed flow c_1 = 2.89 for symmetric heating and 0.408 for asymmetric heating. The developing flow form is based on heat transfer from a vertical plate [11] and the coefficient (0.73) is obtained by regression of data in vertical channels [1,3,4]. The objective of the present work is to develop an analogous composite relationship for inclined channels with constant heat flux boundaries. The work was motivated by an interest in cooling of solar photovoltaic panels via buoyancy driven flow of ambient air in a channel beneath the solar panel. Thus, the specific interest is the top surface temperature.

Much of the prior work on inclined channels considers isothermal boundaries [12–17]. Data for channels with constant heat flux boundaries are more limited in scope. Manca et al. [18] and Bianco et al. [19] performed experiments for symmetric and asymmetric heating (top wall heated) and inclination angles in the range $30^\circ \leqslant \phi \leqslant 90^\circ$. Both studies provide simple curve fits of the data in a similar format:

$$\log(\overline{\mathrm{Nu}}) = a_1 + a_2\log(\mathrm{Ra}''\sin\phi) + a_3[\log(\mathrm{Ra}''\sin\phi)]^2$$
$$\left\{\begin{array}{l} 10 \le \mathrm{Ra}'' \le 10^5 \\ 30^\circ \le \phi \le 90^\circ \end{array}\right\} [18], \quad (2)$$

$$\log(\overline{\mathrm{Nu}}) = a_4 + a_5\log[\mathrm{Ra}''\sin(\phi - 2.7)] + a_6\{\log[\mathrm{Ra}''\sin(\phi - 2.7)]\}^2$$
$$\left\{\begin{array}{l} 10 \le \mathrm{Ra} \le 10^5 \\ 30^\circ \le \phi \le 90^\circ \end{array}\right\} [19]. \quad (3)$$

In Eqs. (2) and (3), the Nusselt number is based on the average top wall temperature, and the constants $a_1 \ldots a_6$ depend on the heating mode (symmetric, asymmetric). Manca and Nardini [20] provide a composite relation for inclined channels in a format similar to that of Bar-Cohen and Rosenhow [5]:

$$\overline{\mathrm{Nu}}_{t,b} = \{[c_1(\mathrm{Ra}''\sin\phi)^{1/2}]^{-2.3} + [c_2(\mathrm{Ra}''\sin''\phi)^{1/5}]^{-2.3}\}^{-1/2.3}$$
$$\left\{\begin{array}{l} 10 \le \mathrm{Ra}'' \le 10^5 \\ 5^\circ \le \phi \le 90^\circ \\ c_2 = 0.81 \text{ for symmetric heating} \\ c_2 = 1.41 \text{ for asymmetric heating} \end{array}\right\} \quad (4)$$

In Eq. (4), the Nusselt number is based on the average temperature of both surfaces. Consequently, the relationship is not applicable to determine the temperature of the top wall.

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Nomenclature

A	coefficient, Eq. (38)
B	coefficient, Eq. (39)
b	coefficient, Eq. (23)
$C_1 \ldots C_4$	coefficients, Eq. (29)
c_p	specific heat (kJ/kg K)
D_h	hydraulic diameter (m)
g	gravitational acceleration (m/s^2)
k	thermal conductivity (W/m K)
L	channel length (m)
$\dot{m}$	mass flow rate (kg/s)
$\overline{\mathrm{Nu}}$	average channel Nusselt number, $q_1''S/k(\overline{T_1} - T_o)$
$\mathrm{Nu}_{1/2}$	channel Nusselt number at the heated wall mid-height, $q_1''S/k(T_{1,L/2} - T_o)$
$\overline{\mathrm{Nu}}_{t,b}$	Nusselt number, based on the average surface temperature for both walls
p	pressure (kg/m s^2)
P	modified pressure, $p - p_e + \rho_o g \sin\phi(x - L)$ (kg/m s^2)
P^*	dimensionless pressure, $P/\rho_o \beta\theta_{be} gL \sin\phi$
Pr	Prandtl number, ν/α
q''	heat flux (W/m^2)
Ra$''$	modified channel Rayleigh number, $g\beta q_1'' S^5/\alpha\nu kL$
r	heat flux ratio, q_2''/q_1''
S	channel spacing (m)
T	temperature (K)
u	streamwise velocity (m/s)
u_o	characteristic fluid velocity (m/s)
u^*	dimensionless streamwise velocity
v	transverse velocity (m/s)
x, y	coordinates (m)
x^*, y^*	dimensionless coordinates

Greek symbols

α	thermal diffusivity (m^2/s)
β	thermal expansion coefficient (K^{-1})
θ	temperature difference ($T - T_o$) (K)
θ^*	dimensionless temperature difference, $\theta k/q_1''S$
μ	viscosity (kg/m s)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ	shear stress (kg/m s^2)
ϕ	inclination angle (deg)
γ	coefficient in Eq. (30), $b(S/L)\cot\phi$
κ	coefficient in Eq. (30), $\sqrt{\frac{1+r}{Ra''\sin\phi}}$
λ	coefficient in Eq. (28), $(\lambda \equiv \frac{1}{4} bRa''\sin\phi)^{1/4}$
ξ	constant, Eq. (21)

Subscripts

1	top wall
2	bottom wall
be	bulk exit
c	cross section
e	channel exit
i	channel inlet
o	reference
s	surface
w	wall

Brinkworth et al. [21–23] propose an approximate model for free convection in inclined channels with top heating. The approach estimates the mass flow rate in the channel from a force balance and then assumes forced convection. The second step essentially decouples the velocity and the temperature fields and assumes the velocity profile is always symmetrical. The adequacy of this stage is therefore questionable over a wide range of Rayleigh numbers.

The composite relation for the average channel Nusselt number developed in the present study provides an accurate estimate of the top wall temperature for buoyant air flow through inclined channels with uniform heat flux boundaries. The form of the composite relationship follows that presented in [5] with the relevant dimensionless parameters for an inclined channel. The fully developed Nusselt–Rayleigh number correlation is based on an analytical solution of the two-dimensional governing conservation equations. The analytical approach follows the earlier work by Aung [2] for vertical channels with isoflux walls. The coefficients required for the composite relationship are derived from a regression analysis of the analytical solution provided in this study and the prior data for developing flow as given by Eq. (2). The correlation takes into account the wall heat flux ratio and reduces successfully to the prior relation for vertical channels [5]. The suggested composite relation is valid for $1 \le \mathrm{Ra}'' < 10^5$ and $30^\circ \le \phi \le 90^\circ$.

2. Fully developed flow

The analytical solution for the average temperature of the top surface of the channel and the Nusselt number based on this temperature is developed for an inclined channel of length L and spacing S (Fig. 1). Uniform heat flux is applied at each surface. The channel width is assumed to be much larger that the channel spacing and thus the problem is two-dimensional. For low Rayleigh numbers, or equivalently for small spacings, fully developed flow conditions are achieved. For aspect ratios, L/S, greater than 10, the streamwise momentum and thermal diffusion are negligible [9,12,24] and thus the conservation equations, subject to the Boussinesq approximation, are

$$0 = -\frac{1}{\rho_o}\frac{\partial P}{\partial x} + \nu\frac{d^2u}{dy^2} + (g\beta\sin\phi)\theta, \quad (5)$$

$$0 = -\frac{1}{\rho_o}\frac{\partial P}{\partial y} + (g\beta\cos\phi)\theta, \quad (6)$$

$$u\frac{\partial\theta}{\partial x} = \alpha\frac{\partial^2\theta}{\partial y^2}, \quad (7)$$

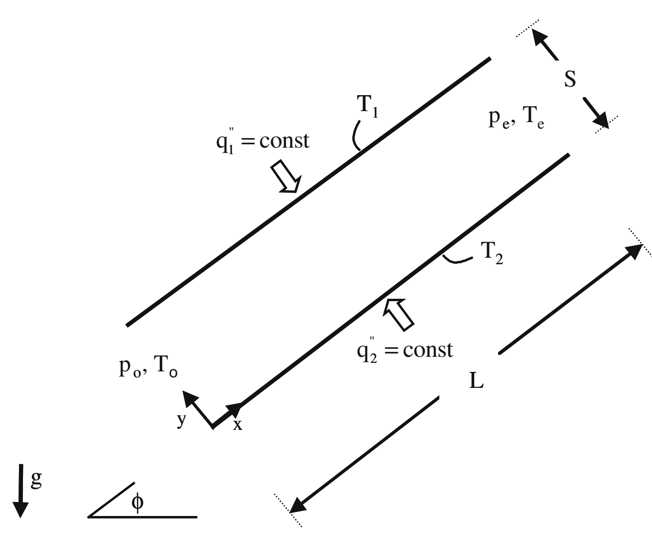

Fig. 1. Inclined channel with constant heat flux boundaries.

where P is the modified pressure, $P \equiv p - p_e + \rho_o g \sin\phi(x - L)$, and $\theta \equiv T - T_o$. The boundary conditions at the channel walls are

$$\text{at } y = 0, \quad u = 0, \quad k\frac{\partial T}{\partial y} = -q_2'', \tag{8a}$$

$$\text{at } y = S, \quad u = 0, \quad k\frac{\partial T}{\partial y} = q_1''. \tag{8b}$$

Ambient temperature and pressure are assumed at the inlet:

$$\text{at } x = 0, \quad T = T_o, \quad P = 0. \tag{8c}$$

At the exit,

$$\text{at } x = L, \quad P = 0. \tag{8d}$$

The governing equations and boundary conditions are non-dimensionalized using the following length, velocity, pressure, and temperature scales:

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{S}; \quad u^* = \frac{u}{u_o}; \quad P^* = \frac{P}{\rho_o\beta\theta_{be}gL\sin\phi}; \quad \theta^* = \frac{\theta}{q_1''S/k} \tag{9}$$

where u_o is a characteristic velocity, and θ_{be} is the difference between the mean (bulk) exit fluid temperature, T_{be}, and the ambient temperature, T_o. The values of u_o and θ_{be} are determined from force and energy balances in the channel. The balance of buoyancy and friction forces is given by

$$A_c\Delta P = A_s\tau_w. \tag{10}$$

The hydraulic diameter, wall shear stress and pressure difference are scaled as

$$D_h = \frac{4A_c}{A_s/L} = 2S; \quad \tau_w = \mu\frac{u_o}{D_h}; \quad \Delta P = \rho_o L(g\beta\sin\phi)\theta_{be}. \tag{11}$$

The overall energy balance is

$$\rho_o u_o S c_p \theta_{be} = q_1''(1 + r)L, \tag{12}$$

where $r \equiv q_2''/q_1''$ is the ratio of heat flux at the channel walls. Combining Eqs. (10)–(12) yields expressions for u_o and θ_{be}:

$$u_o = \frac{\alpha}{S}(S/L)^{-1}\sqrt{\text{Ra}''\sin\phi(1 + r)}, \tag{13}$$

$$\theta_{be} \cong \frac{q_1''S}{k}\sqrt{\frac{1 + r}{\text{Ra}''\sin\phi}}. \tag{14}$$

Using these scales, the governing equations and boundary conditions expressed in dimensionless form are

$$0 = -\frac{\partial P^*}{\partial x^*} + \frac{d^2u^*}{dy^{*2}} + \sqrt{\frac{\text{Ra}''\sin\phi}{1 + r}}\theta^*, \tag{15}$$

$$0 = -\frac{\partial P^*}{\partial y^*} + \sqrt{\frac{\text{Ra}''\sin\phi}{1 + r}}[(S/L)\cot\phi]\theta^*, \tag{16}$$

$$u^*\frac{\partial\theta^*}{\partial x^*} = \frac{1}{\sqrt{\text{Ra}''\sin\phi(1 + r)}}\frac{\partial^2\theta^*}{\partial y^{*2}}, \tag{17}$$

$$\text{at } y^* = 0, \quad u^* = 0, \quad \frac{\partial\theta^*}{\partial y^*} = -r, \tag{18a}$$

$$\text{at } y^* = 1, \quad u^* = 0, \quad \frac{\partial\theta^*}{\partial y^*} = 1, \tag{18b}$$

$$\text{at } x^* = 0, \quad \theta^* = 0, \quad P^* = 0, \tag{18c}$$

$$\text{at } x^* = 1, \quad P^* = 0. \tag{18d}$$

The dimensionless equations lead to an expectation that the Nusselt number may depend on three dimensionless groups:

$$\text{Nu} = \text{Nu}(\text{Ra}''\sin\phi, r, (L/S)\cot\phi). \tag{19}$$

In agreement with prior analysis [2,5], Eq. (19) reduces to $\text{Nu} = \text{Nu}(\text{Ra}'')$ for vertical channels ($\phi = 90°$) with either symmetric ($r = 1$) or asymmetric heating ($r = 0$). The present work shows that the Nusselt number does not depend on $(L/S)\cot\phi$.

An analytical solution for Eqs. (15)–(18) is derived using a technique similar to that presented by Aung [2] for free convection in a vertical channel. Integrating the energy equation (17) from $y^* = 0$ to 1, and applying the constant heat flux boundary conditions yields

$$\int_0^1 u^*(y^*)\frac{\partial\theta^*}{\partial x^*}dy^* = \sqrt{\frac{1 + r}{\text{Ra}''\sin\phi}}. \tag{20}$$

Thus, $\frac{\partial\theta^*}{\partial x^*} = f_1(y^*)$, and the general solution for the temperature field is formulated as

$$\theta^* = f_1(y^*)(x^* - \xi) + f_2(y^*). \tag{21}$$

Substitution of Eq. (21) into Eq. (17) yields

$$\underbrace{\sqrt{\text{Ra}''\sin\phi(1 + r)}u^*(y^*)f_1(y^*)}_{\text{function of } y^* \text{ only}} = f_1''(y^*)(x^* - \xi) + f_2''(y^*), \tag{22}$$

which is valid only if $f_1''(y^*) = 0$, i.e., $f_1(y^*) = ay^* + b$. The uniform heat flux boundary conditions dictate that $a = 0$. Thus, the expressions for the temperature distribution and the energy equation reduce to

$$\theta^* = b(x^* - \xi) + f_2(y'') \tag{23}$$

and

$$f_2''(y^*) = b\sqrt{\text{Ra}''\sin\phi(1 + r)}u^*. \tag{24}$$

To determine $f_2(y^*)$ and u^*, a second relation is required and is derived from conservation of momentum. First, the temperature profile, Eq. (23), is substituted into the conservation of momentum Eqs. (15) and (16). The pressure terms are eliminated by differentiating Eq. (15) with respect to y^* and Eq. (16) with respect to x^* and combining the two equations. The result is

$$f_2'(y^*) = b(S/L)\cot\phi - \sqrt{\frac{1 + r}{\text{Ra}''\sin\phi}}\frac{d^3u^*}{dy^{*3}}. \tag{25}$$

Thus,

$$f_2(y^*) = b(S/L)\cot\phi y^* - \sqrt{\frac{1 + r}{\text{Ra}''\sin\phi}}\frac{d^2u^*}{dy^{*2}} + c. \tag{26}$$

Differentiating Eq. (25) with respect to y^* and equating it to Eq. (24) yields a fourth order ODE for the velocity field:

$$\frac{d^4u^*}{dy^{*4}} + b\text{Ra}''\sin\phi u^* = 0. \tag{27}$$

The solution is

$$u^*(y^*) = e^{\lambda y^*}(C_1\cos\lambda y^* + C_2\sin\lambda y^*) + e^{-\lambda y^*}(C_3\cos\lambda y^* + C_4\sin\lambda y^*), \tag{28}$$

where $\lambda^4 \equiv \frac{1}{4}b\text{Ra}''\sin\phi$ and $C_1 \ldots C_4$ are constants.

Substitution of Eqs. (26) and (28) into Eq. (23) provides the temperature field:

$$\theta^*(x^*, y^*) = b(x^* - \xi) + b(S/L)\cot\phi y^* + \sqrt{b(1 + r)}[e^{\lambda y^*}(C_1\sin\lambda y^* - C_2\cos\lambda y^*) - e^{-\lambda y^*}(C_3\sin\lambda y^* - C_4\cos\lambda y^*)], \tag{29}$$

where the constant c from Eq. (26) is absorbed into the coefficient ξ. The constants $C_1 \ldots C_4$ are determined by applying the boundary conditions (18a) and (18b).

$$C_1 = \frac{\sin\lambda}{2\kappa\lambda^3}\frac{[\gamma - 1 + 2e^{\lambda}\sin\lambda(\gamma + r) + e^{2\lambda}(1-\gamma)]}{[e^{-\lambda} - 2e^{\lambda}(1 + 2\kappa - 2\kappa\cos^2\lambda) + e^{3\lambda}]}, \tag{30a}$$

$$C_2 = \frac{C_1[2 - \cot\lambda(1 - e^{2\lambda})] + (\gamma + r)/2\kappa\lambda^3}{1 - e^{2\lambda}}, \tag{30b}$$

$$C_3 = -C_1, \tag{30c}$$

$$C_4 = C_1\cot\lambda(1 - e^{2\lambda}) - C_2 e^{2\lambda}, \tag{30d}$$

where $\kappa = \sqrt{\frac{1+r}{\mathrm{Ra}''\sin\phi}}$ and $\gamma = b(S/L)\cot\phi$.

The constant b is evaluated using the fluid temperature at the outlet of the channel. The dimensionless mean outlet air temperature is

$$\theta^*_{\mathrm{be}} = \frac{\int_0^1 u^*(y^*)\theta^*(x^* = 1)dy^*}{\int_0^1 u^*(y^*)dy^*}. \tag{31}$$

By replacing the characteristic velocity, u_{o}, in Eq. (12) with the mean velocity, one can show that the dimensionless mean outlet air temperature is also given by

$$\theta^*_{\mathrm{be}} = \frac{\kappa}{\int_0^1 u^*(y^*)dy^*}. \tag{32}$$

Equating Eqs. (31) and (32) yields

$$\int_0^1 u^*(y^*)\theta^*(x^* = 1)dy^* = \kappa, \tag{33}$$

which after the substitution of the velocity and temperature profiles given in Eqs. (28) and (29) becomes an algebraic equation for b.

Finally, ξ is estimated from the channel exit pressure boundary condition. By integrating Eqs. (15) and (16) with respect to x^* and y^*, respectively, and comparing the terms, an expression for the pressure distribution is determined:

$$P^*(x^*, y^*) = \frac{\gamma}{\kappa}x^*y^* + \frac{b}{2\kappa}x^{*2} - \frac{b\xi}{\kappa}x^* + \frac{\gamma^2}{2b\kappa}y^{*2} - \frac{\gamma\xi}{\kappa}y^* - \frac{\gamma}{b}\frac{du^*}{dy^*}. \tag{34}$$

The dimensionless exit pressure P^*_{e} reduces to zero for the vertical orientation only if $\xi = 1/2$. In general, for inclined channels, $P^*_{\mathrm{e}} \neq 0$. However, for the range of parameters considered here ($1 \le \mathrm{Ra}'' \le 10^2$, $20 \leqslant L/S \leqslant 571$ and $30 \leqslant \phi \leqslant 90°$), the deviation in the exit pressure from zero is negligible ($P^*_{\mathrm{e}} \ll 1$). The average channel Nusselt number based on the top wall temperature is defined as

$$\overline{\mathrm{Nu}} \equiv \frac{q_1''}{\overline{T_1} - T_{\mathrm{o}}}\frac{S}{k} = \frac{1}{\theta^*(x^* = 1/2, y^* = 1)} \quad \text{where } \overline{T_1} = \frac{1}{L}\left(\int_0^L T_1 dx\right). \tag{35}$$

The average and mid-height temperatures are identical for fully developed flow because the temperature distribution is linear.

3. Developing flow

For high Ra'' numbers (large channel spacings), the Nusselt number takes the form of the single plate limit. Because our interest is in the top wall temperature, the relevant configuration is a downward facing plate. For this orientation, the average Nusselt/Rayleigh correlation is [11]

$$\overline{\mathrm{Nu}} = 0.69(\mathrm{Ra}''\sin\phi)^{1/5}. \tag{36}$$

Following this format, the average channel Nusselt number for inclined channels at high Ra'' may be formulated as

$$\overline{\mathrm{Nu}} = B(\mathrm{Ra}''\sin\phi)^{1/5}, \tag{37}$$

where B is anticipated to greater than 0.69 due to the chimney effect in channels [1,3,4]. In the present work the value of B is determined from a regression analysis, as described in the following section.

4. Composite relation

The composite relation is obtained following the procedure suggested by Churchill and Usagi [25]. The correlation between the Nusselt and the Rayleigh number are constructed by interpolation between the asymptotic relations for high and low Rayleigh numbers. The average Nusselt number can be expressed in terms of $\mathrm{Ra}''\sin\phi$ and r. In a format similar to Eq. (1),

$$\overline{\mathrm{Nu}} = \{\overbrace{[A(r)(\mathrm{Ra}''\sin\phi)^{1/2}]^{-n}}^{\text{fully developed flow}} + \overbrace{[B(\mathrm{Ra}''\sin\phi)^{1/5}]^{-n}}^{\text{developing flow}}\}^{-1/n}. \tag{38}$$

The values of A, B, and n are obtained by a regression analysis of Nusselt numbers obtained from Eqs. (29) and (35) for fully developed flow in the range $1 < \mathrm{Ra}''\sin\phi < 10$ (where empirical data are not available) and for developing flow from Eq. (2) in the range $10 < \mathrm{Ra}''\sin\phi < 10^5$. The best fit is found for $n = 2$, $B = 0.78$, and $A = 0.28$ for symmetric heating and $A = 0.4$ for asymmetric heating. The coefficients of determination are $R^2 = 0.9920$ and $R^2 = 0.9939$ for symmetric and asymmetric heating, respectively. In the case of vertical channels, Bar-Cohen and Rosenhow [5] provide values for the coefficient A only for symmetric and asymmetric heating (see Eq. (1)). Applying their model for fully developed flow one can show that $A(r) \propto 1/\sqrt{1+r}$ for any r. Therefore, based on the present regression analysis, we suggest using $A(r) = 0.4/\sqrt{1+r}$.

5. Results

Predictions of the average channel Nusselt number as a function of $\mathrm{Ra}''\sin\phi$ for symmetric ($q_2'' = q_1''$, $r = 1$) and asymmetrical heating ($q_2'' = 0$, $r = 0$) are presented in Fig. 2a and b for $\mathrm{Ra}'' \le 10^5$ and $30° \le \phi \le 90°$. Using the analytical solution for fully developed flow, the aspect ratio, L/S was varied from 20 to 571. For symmetric heating (Fig. 2a), the results are compared to the correlations of Manca et al. [18] and Bar-Cohen and Rosenhow [5]. For asymmetric heating (Fig. 2b), the results are also compared to the correlation of Webb and Hill [7]. In these figures, the Bar-Cohen and Rosenhow [5] and Webb and Hill [7] expressions obtained for vertical channels are modified by replacing g with $g\sin\phi$. There is an excellent agreement of the present analytical predictions for $\mathrm{Ra}'' \le 10^2$ to the modified correlations for vertical channels and data for inclined channels. The agreement between the Bar-Cohen and Rosenhow modified correlation and the analytical solution at low Ra'' (<10) suggests the difference between temperature of the top wall at mid-height and the average temperature along the wall is negligible within this range. At Ra'' > 100, the average Nusselt number is higher than the mid-height Nusselt number. Therefore, for this range, the predictions of Bar-Cohen and Rosenhow falls slightly below the suggested relation. For a given heating mode ($r = 0$ or $r = 1$), the Nusselt number depends only on $\mathrm{Ra}''\sin\phi$. The influence of the inclination angle is felt only in the streamwise direction (i.e., the effect of the dimensionless parameter $(L/S)\cot\phi$ on the average Nusselt number is negligible) and the form of equation (38) is justified. The Nusselt number therefore increases with the inclination angle as $\sin\phi^{1/2}$ for fully developed flow and $\sin\phi^{1/5}$ for developing flow. The results are well represented by

$$\overline{\mathrm{Nu}} = \left[\frac{6.25(1+r)}{\mathrm{Ra}''\sin\phi} + \frac{1.64}{(\mathrm{Ra}''\sin\phi)^{2/5}}\right]^{-1/2} \quad \left\{\begin{matrix} 1 \le \mathrm{Ra}'' \le 10^5 \\ 30° \le \phi \le 90° \end{matrix}\right\}. \tag{39}$$

To illustrate the effect of the heat flux ratio, r, on the Nusselt number, Eq. (39) is plotted in Fig. 3 for $r = 0$, 1/3, 2/3, and 1. For $\mathrm{Ra}'' \le 10^2$, the top wall temperature is linked to the fluid bulk temperature. The latter increases with the total heat rate of the channel,

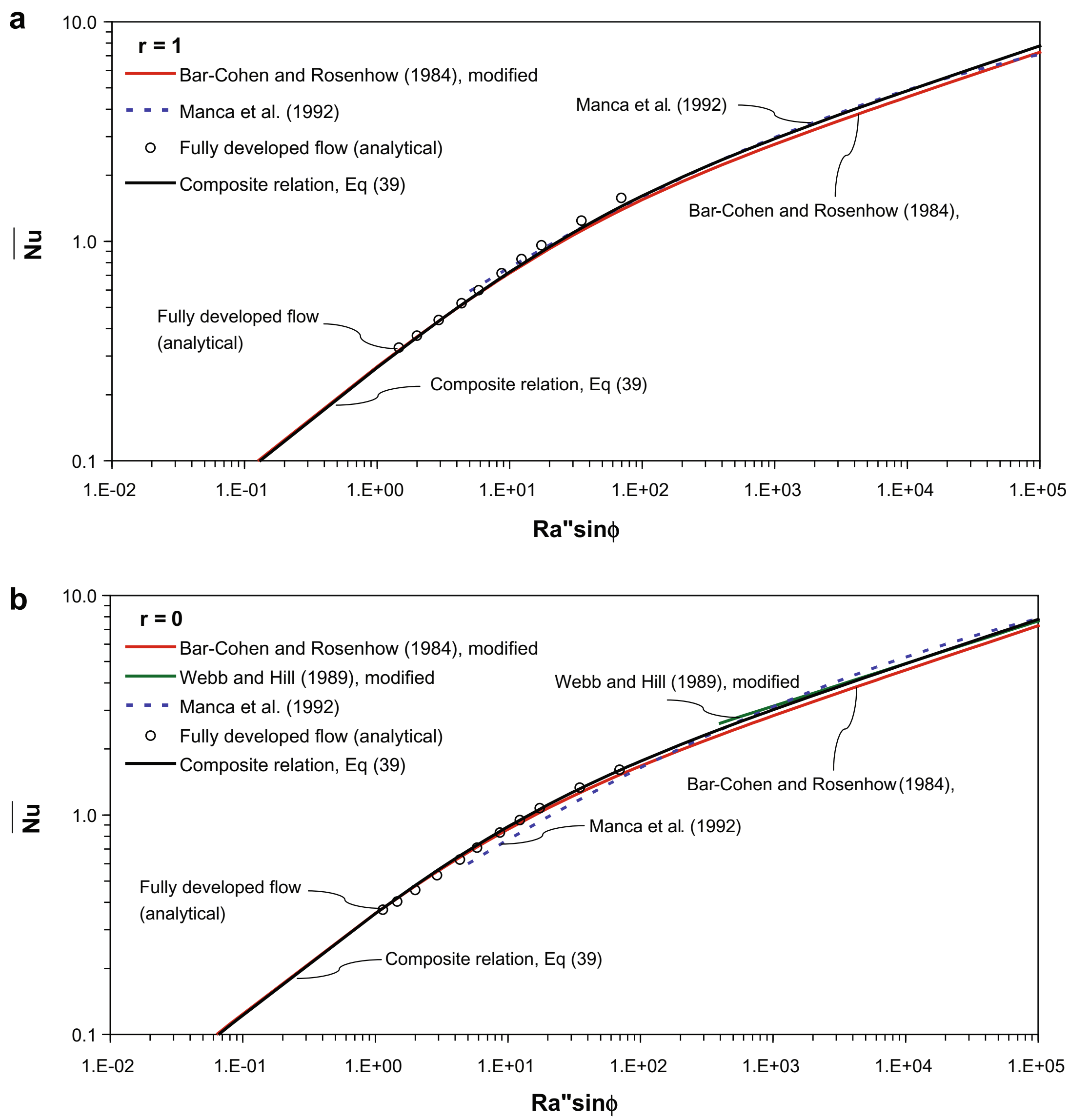

Fig. 2. Nusselt number versus Rayleigh number for (a) symmetric heating and (b) asymmetric heating.

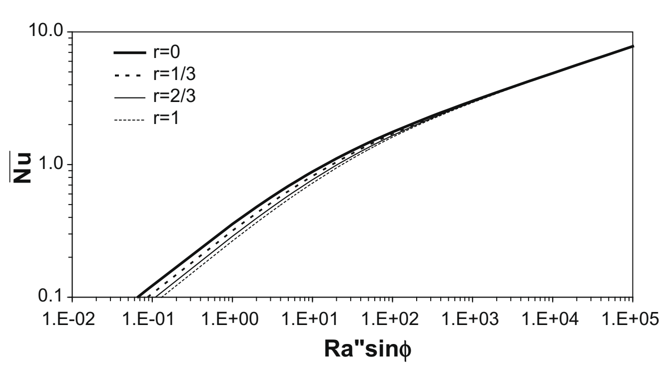

Fig. 3. Nusselt number versus Rayleigh number for heat flux ratios $r = 0$, 1/3, 2/3, and 1.

$q''_1(1 + r)$, and therefore Nu decreases with r. For $Ra'' \leq 10^1$, the flow is developing and heat transfer at the top wall is independent of the bottom wall heating condition.

6. Conclusion

This paper provides a physically based correlation for natural convection of air in open-ended inclined channels when both boundaries are subject to a uniform heat flux with either symmetric or asymmetric heating. A composite relation for the average channel Nusselt number based on the top surface temperature is presented in Eq. (39). This correlation was developed with the help of analytical analysis at low Rayleigh numbers and available empirical data at high Rayleigh numbers. The suggested correlation agrees very well with available data. For either heating mode, the Nusselt number depends only on the modified channel Rayleigh number, multiplied by the gravity component at the streamwise direction. This finding is analogous to the result of Azevedo and Sparrow [12] who tested channels with one or two isothermal walls.

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